fusion current shifts toward $\xi_1 = 1$ at lower values of the Reynolds number (Fig. 2). The surface smoothing factor $c = \left(\dfrac{dU_s}{dn}\Big|_{Y=1}\right)_{max} / \left(\dfrac{dU_s}{dn}\Big|_{Y=1}\right)_{min}$ increases with higher values of the Reynolds number: $c = 1.2$ when $N_{Re} = 20$ and $c = 2.1$ when $N_{Re} = 100$.

Therefore, a dissolving surface is smoothed most intensely prior to formation of a vortex in a surface cavity. The roughness amplitude changes during dissolution and, therefore, the optimum conditions for smoothing will be ensured only by maintenance of a continuously vortex-free mode of streamlining.

The results of this study can be utilized in setting up the finish treatment in chemical manufacturing processes, in electrochemical polishing, and in other technologies where formation of the roughness microprofile of surfaces is largely influenced by the hydrodynamics of their streamlining.

NOTATION

x, longitudinal coordinate; y, normal coordinate; ξ, η, Y, dimensionless coordinates; h_0, mean channel half-width; h_1, channel half-width; h, dimensionless channel half-width; λ, roughness wavelength; A, roughness amplitude; ψ, flow function; Ψ, dimensionless flow function; U_{max}, maximum velocity in the channel; u, v, u_s, velocity components along the x axis, the y axis, and along the wall contour, respectively; U, V, U_s, dimensionless velocity components; ν, kinematic viscosity; $A_{2k,n}$, coefficients of the series expansion; N_{Re}, Reynolds number; c, surface smoothing factor; and ε, δ, parameters.

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GROWTH OF PARAFFIN DEPOSITS ON THE PIPE SURFACE

IN A STREAM OF OIL

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In describing the kinetics of the growth of a layer, it was taken that the solidification temperature is a variable and depends on the paraffin concentration in the stream of oil in a pipe.

When the soil temperature in the vicinity of a pipe transporting paraffinous crude drops, the layers of the liquid near the pipe surface solidify and lose their mobility. These phenomena are connected with the appearance of paraffin crystals in the near-wall layers of the liquid; these crystals form the structure within which the liquid is retarded. The temperature at which the near-wall layers of the liquid lose their mobility is called the solidification point. Experiments showed that the solidification point depends on the paraffin concentration in the stream [1]. This circumstance must be taken into account in describing the kinetics of the growth of the layer of deposits on the inner pipe surface.

We will examine turbulent flow of crude oil in the pipe. The processes of impulse, heat and mass transfer in a turbulent flow of liquid in a pipe are such that the local values of speed, temperature, and concentration almost over the entire cross section of the flow differ little from their mean values over the cross section of the pipe. We may therefore neglect the nonuniformity of the profiles of the sought values and describe the process with the aid of the mean parameters over the cross section of the pipe which depend only on the time t and the distance along the x axis. Then the equations describing the process are univariate.

We will proceed from a two-layer model of turbulent heat and mass transfer and assume that between the boundary of the flow core and the surface of the layer of deposits there is a zone in which the concentration and the temperature change from θ and T in the core to θ_w and T_w, the values of these parameters at the boundary of the layer of deposits.

It is assumed that the parameters θ_w and T_w are mutually correlated: $T_w = T_w(\theta_w)$. The simplest form of such a dependence for paraffin dissolved in crude oil is the following [1]:

$$T_w = a_1 + a_2 \lg \frac{\theta_w}{\rho}.$$

(1)

The accepted arrangement of mass transfer makes it possible to introduce the mass transfer coefficient β and to represent the flow of matter to the surface of the layer of deposits in the form

$$j_w = 2\pi\beta\delta(\theta - \theta_w) - 2\pi\delta\theta_w \frac{\partial\delta}{\partial t}.$$

(2)

The second component on the right-hand side of this expression is determined by the growth of the layer on whose surface new paraffin particles appear every instant.

The stream of matter j_w serves to increase the thickness of the layer of deposits, consequently,

$$(\rho_p - \theta_w) \frac{\partial\delta}{\partial t} = \beta(\theta_w - \theta).$$

(3)

We write the equation of mass transfer in the flow core:

$$\frac{\partial\theta}{\partial t} + U\frac{\partial\theta}{\partial x} = 2\left(\frac{\partial\delta}{\partial t} + \beta\right)\frac{\theta_w - \theta}{\delta}.$$

(4)

This equation can be easily derived, taking expression (3) into account, from the condition of constant quantity of paraffin in the system flow core—layer of deposits. Equations (3) and (4) describe the mass transfer in the examined system but they do not constitute a closed system. The point is that the value of θ_w is variable and depends on the solidification point T_w. In order to close the system, we must add equations describing heat exchange to Eqs. (3) and (4). One such equation is Stefan' condition which in the case under examination has the form

$$(\rho_p - \theta_w)l\frac{\partial\delta}{\partial t} = \frac{T_M - T_w}{\delta\left(\frac{1}{\lambda_p}\ln\frac{R_i}{\delta} + \frac{1}{\lambda_{so}G}\right)} + \alpha(T - T_w).$$

(5)

Here the magnitude G(t) characterizes the thermal resistance of the distorted soil layer. The expression for calculating G(t) is the following [2]:

$$G = \max\left\{\frac{1}{\alpha_0}; \frac{R - 1}{R\ln R - R + 1}\right\};$$

$$\alpha_0 = \ln(H/R_i + \sqrt{(H/R_i)^2 - 1)}.$$

(6)

Here R(t) is the outer radius of the distorted soil layer which is determined from the equation [2]:

$$\frac{dR}{dt} = \frac{\varkappa_{so}}{R_i^2} \frac{\ln R + 1/R - 1}{R\ln R(1 + 1/R + 1/R^2)/6 - R/4 + 1/(4R)}.$$

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$$R|_{t=0} = 1. \tag{7}$$

The following is the equation of heat transfer to the flow core:

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} = 2\left(\frac{\partial \delta}{\partial t} + \frac{\alpha}{\rho c_{\text{H}}}\right)\frac{T_w - T}{\delta} + \frac{giU}{c_{\text{H}}E}. \tag{8}$$

The expression for the hydraulic gradient i has the form [3]

$$i = \frac{Q^2}{4\pi^2 g R_i^5 (1.8 \lg \text{Re} - 1.5)^2};$$

$$Q = \pi R_i^2 U; \quad \text{Re} = 2UR_i/\nu; \quad 3\cdot10^3 \leqslant \text{Re} \leqslant 3\cdot10^6. \tag{9}$$

The above univariate equation contains the mass-transfer coefficient β and of heat-transfer coefficient α. These coefficients depend on the determining parameters of the process, and if such dependences are known, then various problems can be solved with the aid of these equations.

Within the framework of the adopted two-layer model of turbulent flow, we can easily write the expressions for the coefficients β and α:

$$\beta = \frac{D}{\Delta_D}; \quad \alpha = \frac{\lambda}{\Delta_\lambda}. \tag{10}$$

The thicknesses of the diffusion and thermal layers are part of the dynamic layer and are equal to [4]

$$\Delta_D = \Delta_\mu \text{Sc}^{-\frac{1}{4}}; \quad \Delta_\lambda = \Delta_\mu \text{Pr}^{-\frac{1}{4}};$$

$$\text{Sc} = \nu/D; \quad \text{Pr} = \nu/\varkappa; \quad \varkappa = \lambda/(\rho c_{\text{H}});$$

$$\Delta_\mu = 11.5\nu/u_*; \quad u_* = U\sqrt{\zeta/8}. \tag{11}$$

It follows from (10) and (11) that

$$\beta = 0.031\sqrt{\zeta}\,\text{Sc}^{-0.75}U; \quad \alpha = 0.031\,\rho c_{\text{H}}\sqrt{\zeta}\,\text{Pr}^{-0.75}U. \tag{12}$$

It should be pointed out that expressions (9), (12) are correct for crude oils described by the model of a Newtonian liquid. For crude oils with substantially non-Newtonian properties, these expressions may be adopted in accordance with [5, 6].

Expressions (10) are correct on the assumption that the thicknesses of diffusion distortion and of thermal distortion in the flow are much larger than Δ_D and Δ_λ. We will estimate the times for which expressions (10) are correct. The lengths of the zones of thermal and diffusion distortions increase with time proportionally to $\sqrt{\varkappa t}$ and $\sqrt{Dt}$, respectively. Hence follows the estimate $t \gg \Delta_D^2/D$; $t \gg \Delta_\lambda^2/\varkappa$. It is easy to verify that the characteristic times Δ_D^2/D and $\Delta_\lambda^2/\varkappa$ in a developed turbulent flow of liquid are of the order of magnitude of 1 h. The durations of the process that are of interest are counted in days, therefore practically for the entire duration of the process, expressions (10) may be considered correct. However, at the very first stage of the process of deposition of paraffin and of the growth of the layer, there are a number of peculiarities which are not taken into account in the univariate model. For instance, concentration and temperature at the layer boundary are interrelated, it is therefore not clear which values have to be taken for these magnitudes as initial conditions for describing the process within the framework of the univariate model. Here we suggest the following approach. Let $t < \Delta_D^2/D$ or $\Delta_\lambda^2/\varkappa$. With such times, the processes of heat and mass exchange penetrate deeply into the laminar sublayer. To describe the process at these instants, we formulate the following problem of Stefan type:

$$\frac{\partial T_1}{\partial t} = \varkappa\frac{\partial^2 T_1}{\partial y^2}; \quad \frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial y^2}; \quad y_0(t) \leqslant y < \infty;$$

$$\frac{\partial T_2}{\partial t} = \varkappa_{\text{p}}\frac{\partial^2 T_2}{\partial y^2}; \quad c = \rho_{\text{p}}; \quad 0 \leqslant y \leqslant y_0(t);$$

$$T_1|_{t=0} = T_1|_{y=\infty} = T_0; \quad T_1|_{y=y_0} = T_2|_{y=y_0} = T_w;$$

$$T_2|_{y=0} = T_{\text{M}}; \quad c|_{t=0} = c|_{y=\infty} = \theta; \quad c|_{y=y_0} = \theta_w;$$
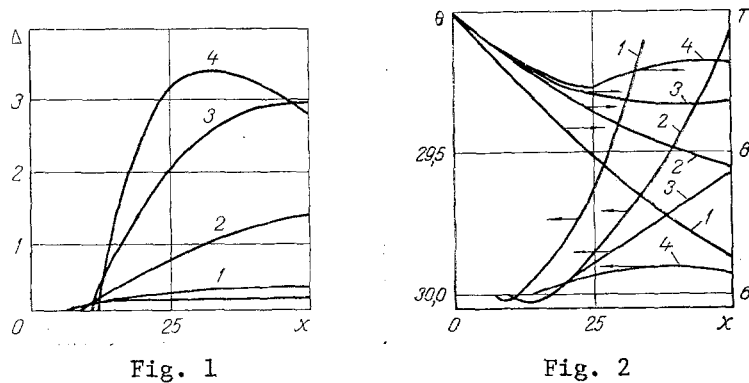
Fig. 1   Fig. 2

Fig. 1. Change in the thickness of the paraffin lay-
er $\Delta = R_1 - \delta$ along the pipe at different instances: 1)
125 h; 2) 625 h; 3) 2500 h; 4) 5000 h. Δ, cm; x, km.

Fig. 2. Change of the paraffin concentration θ in the
stream of crude oil and of temperature T along the pipe
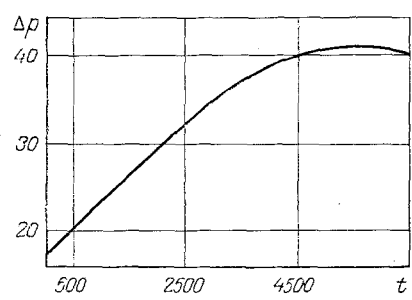at different instants: 1) t = 125 h; 2) 625 h; 3) 2500
h; 4) 5000 h.

Fig. 3. Change in the losses
of head during pumping Δp
with time. Δp, atm; t, h.

$$(\rho_p - \theta_w)\,\dot{y}_0 = D \frac{\partial c}{\partial y}\bigg|_{y=y_0};$$

$$\rho_p l \dot{y}_0 = \left(\lambda_p \frac{\partial T_2}{\partial y} - \lambda \frac{\partial T_1}{\partial y}\right)_{y=y_0}.$$

(13)

The correlation between T_w and θ_w is given by expression (1). In formulating this problem
we disregarded the convective component of heat and mass transfer because the densities of
crude oil and of pollutant paraffin are almost equal.

Problem (13) has a self-similar solution depending on the Boltzmann variable:

$$\frac{T_1 - T_0}{T_w - T_0} = \frac{\mathrm{erfc}\,(y/2\sqrt{\varkappa t})}{\mathrm{erfc}\,(p/\sqrt{\varkappa})};$$

$$\frac{T_2 - T_M}{T_w - T_M} = \frac{\mathrm{erf}\,(y/2\sqrt{\varkappa t})}{\mathrm{erf}\,(p/\sqrt{\varkappa})};$$

$$\frac{c - \theta}{\theta_w - \theta} = \frac{\mathrm{erfc}\,(y/2\sqrt{Dt})}{\mathrm{erfc}\,(p/\sqrt{D})}; \qquad y_0 = p\sqrt{t}.$$

(14)

Using the conditions of equilibrium at the boundary of phase transition, we obtain trans-
cendental equations for determining T_w, θ_w, and p:

$$\rho_p p = \sqrt{D/\pi}\,(\theta - \theta_w)\exp\,(-p^2/D)/\mathrm{erfc}\,(p/\sqrt{D});$$

$$\rho_p l p = \lambda_p(T_w - T_M)\exp\,(-p^2/\varkappa_p)/[\sqrt{\pi\varkappa_p}\,\mathrm{erf}\,(p/\sqrt{\varkappa_p})] -$$

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$$- \lambda \left(T_w - T_0 \right) \exp \left(- p^2 / \varkappa \right) / [\sqrt{\pi \varkappa} \, \text{erfc} \, (p / \sqrt{\varkappa})];$$

$$T_w = a_1 + a_2 \lg \frac{\theta_w}{\rho}. \tag{15}$$

The solution of these equations, with a view to the real values of the parameters contained in them, shows that the difference $(T_w - T_M)$ is equal to one hundredth of a degree. Therefore in solving the univariate system of equations we took

$$T_w|_{t=0}; \quad \theta_w|_{t=0} = \rho \cdot 10^{(T_M - a_1)/a_2}. \tag{16}$$

The initial thickness of the layer was taken to be equal to zero. To obtain a numerical solution, we can make a number of simplifications in the initial system of equations (1), (3)-(9). Under real conditions of the process, $\theta_w \ll \rho_p$. From this we obtain easily that $|\delta| \ll \beta$. Approximate calculations show that the heat flux from the crude oil to the soil considerably exceeds the heat content liberated from the paraffin layer in unit time $\rho c \delta \ll \alpha$ and the heat liberated by crystallization. These facts enable us to transform the initial system into a form that is more convenient for numerical calculations:

$$\rho_p \frac{\partial \delta}{\partial t} = \beta \left(\theta_w - \theta \right); \tag{17}$$

$$\delta|_{t=0} = R_i; \tag{18}$$

$$\frac{\partial \theta}{\partial t} + U \frac{\partial \theta}{\partial x} = \frac{2\beta}{\delta} \left(\theta_w - \theta \right); \quad \begin{array}{l} x \leqslant Ut; \\ 0 \leqslant x \leqslant L; \end{array} \tag{19}$$

$$\theta|_{x=0} = \theta_0; \tag{20}$$

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} = \frac{2}{\pi \rho c_{\text{H}} \delta^2} \frac{T_{\text{M}} - T}{\left(\dfrac{1}{\lambda_p} \ln \dfrac{R_i}{\delta} + \dfrac{1}{\lambda_{\text{so}} G} + \dfrac{1}{\alpha \delta} \right)} + \frac{giU}{c_{\text{H}} E}; \quad \begin{array}{l} x \leqslant Ut; \\ 0 \leqslant x \leqslant L; \end{array} \tag{21}$$

$$T|_{x=0} = T_0; \tag{22}$$

$$T_w = T - \frac{T - T_{\text{M}}}{\alpha \delta \left(\dfrac{1}{\lambda_{\text{so}} G} + \dfrac{1}{\lambda_p} \ln \dfrac{R_i}{\delta} \right) + 1}. \tag{23}$$

These equations describe the process of filling the pipes with crude oil and the subsequent operation. For solving the system of univariate equations (17)-(23), a program in FORTRAN-V was prepared and realized on a computer. The calculations were carried out with the following values of the parameters: $\theta_0 = 30$ kg/m^3; $T_0 = 10°C$; $R_i = 0.15$ m; $H = 1.8$ m; $\nu = 0.05$ m^2/h; $c_{\text{H}} = 0.5$ kcal/kg·deg; $\rho_p = 800$ kg/m^3; $l = 50$ kcal/kg; $\lambda_p = 0.5$ kcal/m·h·deg; $\beta = 0.5 \cdot 10^{-2}$ m/h; $a_1 = 114°C$; $a_2 = 28.1°C$; $T_M = -5°C$; $\varkappa_{\text{so}} = 0.00778$ m^2/h; $\lambda_{\text{so}} = 1.8$ kcal/m·h·deg; $L = 50$ km; $Q = 250$ m^3/h.

The results of the calculations are presented in Figs. 1-3. Figure 1 shows the evolution of the paraffin layer on the surface of the pipe. The front edge of the layer moves in the course of time away from the initial cross section on account of the rising temperature of the crude oil during the process of heating of the surrounding soil. At t = 5000 h the front edge is 12 km away from the initial cross section. It is interesting to note that whereas the layer grows monotonically along the pipe when its thickness is small, a maximum appears at some cross section of the pipe when the layer is thick. This is connected with the fact that at considerable thicknesses of the chill zone, the heat-dissipation capacity increases and the temperature of the flow rises, raising the temperature of the layer surface and correspondingly decreasing the intensity of the migration flow of the paraffin. The corresponding phenomena can also be noted in Fig. 2 which shows the concentration profiles of paraffin in the flow and the temperatures of the flow at different instants. Here it may be noted that on the section, where the front edge of the layer becomes blurred, the paraffin concentration in the stream increases. In addition to that, for high values of time (t = 5000 h), there is a minimum concentration corresponding to the maximum thickness of the layer.

The intensity of the temperature drop along the pipe decreases with time because of the heating of the surrounding soil and the increase of the power of the dissipative sources of heat when the thickness of the paraffin layer increases. Beginning at some instant, the power

of the dissipative sources, which increases downstream, balances at some point the heat losses to the soil, and the flow temperature, having passed through its minimum, begins to increase.

Figure 3 shows the change in the losses of head along the pipe in time. In connection with the growth of the paraffin layer, the losses of head in pumping increase from 16.6 atm for a clean pipe to 41 atm for t = 5500 h. After that the losses of head begin to decrease because of the destruction of the paraffin layer on the pipe wall.

NOTATION

θ, T, paraffin concentration and temperature, respectively, in the flow core; θ_w, T_w, concentration and temperature, respectively, on the surface of the layer of deposits; ρ, density of the crude oil; j_w, flow of paraffin to the surface of the layer; β, mass-transfer coefficient; Δ, thickness of the layer of deposits; ρ_p, density of the paraffin; U, mean flow velocity; l, latent heat of crystallization; T_M, initial soil temperature; λ, λ_p, λ_{so}, thermal conductivity of crude oil, paraffin, and soil, respectively; α, heat-transfer coefficient; R_i, pipe radius; c_H, heat capacity of crude oil; g, acceleration of gravity; ν, kinematic viscosity of crude oil; Δ_μ, Δ_D, Δ_λ, thickness of the dynamic, diffusion, and thermal layers, respectively; ζ, coefficient of hydraulic resistance; D, diffusion coefficient; $\varkappa$, thermal diffusivity; T_1, oil temperature; T_2, temperature of the layer of deposits; c, local paraffin concentration; y_0, thickness of the layer; y, distance to the pipe surface; H, depth of the pipe in the ground; T_0, θ_0, temperature and concentration, respectively, in, initial cross section of the pipe; L, length of the pipe; t, time; x, distance in the direction of the axis; δ, radius of the inner surface of the paraffin layer.

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